Int. J. Heat Mass Transfer. Vol. 24, No. 6, pp. 1025–1032, 1981
Printed in Great Britain

0017-9310/81/061025-08 $02.00/0
Pergamon Press Ltd.

FULLY-DEVELOPED NUCLEATE BOILING: OVERLAP OF AREAS OF INFLUENCE AND INTERFERENCE BETWEEN BUBBLE SITES

D. B. R. KENNING and VICTOR H. DEL VALLE M.*
Department of Engineering Science, Oxford University, Oxford OX1 3PJ, U.K.

(*Received* 5 *August* 1980 *and in revised form* 25 *November* 1980)

Abstract—The surface quenching model for boiling heat transfer at low nucleation site densities has been extended to high densities. The enhanced rate of heat transfer due to the increased quenching frequency where the areas of influence of several nucleation sites overlap partly offsets the reduction in boiling area. The degree of overlap has been estimated for random and regular distributions of nucleation sites, and for a random distribution modified by short-range interference between sites. Its effect on heat transfer is not large in the range of practical interest.

Evidence for the inhibition of nucleation by thermal interference from active sites is reviewed. Although interference has little effect on the heat transfer model for a specified number of active sites, it may influence the developed region of the boiling curve by affecting the number of active sites and the formation of vapour patches by bubble coalescence. Relations between the densities of potential and active sites have been derived.

NOMENCLATURE

- A, maximum bubble projected area, $= \pi R_m^2$;
- c, liquid specific heat;
- f, bubble frequency;
- h, radius of inhibited nucleation zone;
- Ja, Jacob number, $= \rho c \Delta T / \lambda \rho_g$;
- k, liquid conductivity;
- K, ratio area of influence to maximum bubble projected area;
- n, density of active nucleation sites;
- n_0, density of potential nucleation sites;
- P, probability (defined as required in text);
- q, total heat flux;
- q_{bi}, boiling flux, isolated bubbles;
- q_c, convective flux;
- q_1, time-averaged quenching flux (1 site);
- q_m, time-averaged quenching flux (m sites, random phase);
- r, radius co-ordinate;
- R, radius area of influence, $= R_m K^{1/2}$;
- R_m, maximum bubble radius;
- s, nearest-neighbour separation;
- ΔT, wall superheat;
- x, ratio actual to nominal boiling area;
- y_m, fraction of area of influence quenched by m sites;
- α, nominal fraction of wall affected by boiling, $= nKA$;
- α_0, potential boiling area fraction, $= n_0 KA$;
- ρ, liquid density;
- ρ_g, vapour density;
- τ, bubble period, $= 1/f$;
- λ, latent heat.

* Present address: Instituto Nacional de Investigaciones Nucleares, Mexico 18, D.F. Mexico.

1. INTRODUCTION

IN SATURATED pool boiling at low heat flux (the 'isolated bubble' regime) some success has been claimed for heat transfer models which express the total heat flux q as the sum of a boiling component q_{bi} and a single-phase convective flux q_c on the fraction $(1 - \alpha)$ of the total surface area unoccupied by boiling [1–4]

$$q = q_{bi} + q_c(1 - \alpha). \tag{1}$$

The isolated boiling flux depends on the active nucleation site density n and the wall superheat ΔT. In the simplest version of the model it is calculated by the 'surface quenching' mechanism: transient conduction from the wall to semi-infinite liquid which is replaced at the bubble frequency f by fresh liquid at the bulk (i.e. saturation) temperature over an area of influence K times the maximum bubble projected area A. The time-averaged quenching flux q_1 on the areas of influence is

$$q_1 = 2(k\rho c f/\pi)^{1/2} \Delta T. \tag{2}$$

The isolated boiling flux is conventionally referred to the total surface area so

$$q_{bi} = q_1 \alpha \tag{3}$$

$$\alpha = nKA = nK(\pi R_m^2). \tag{4}$$

Thus the problem of predicting the boiling curve $q(\Delta T)$ is broken down into the separate steps of predicting n, f and the maximum bubble radius R_m as functions of ΔT. The statistical fluctuations in bubble characteristics are generally ignored and K is assumed constant in the range $2 \leqslant K \leqslant 5$.

This model grossly simplifies the flow and heat transfer in the vicinity of an isolated bubble site but can be refined, for example, by the addition of microlayer

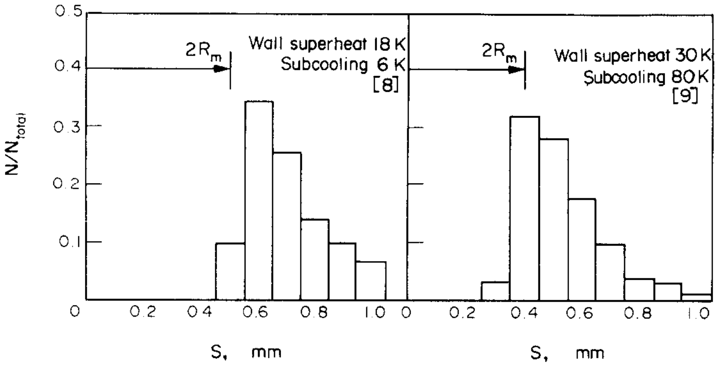

FIG. 1. Distributions of nearest-neighbour distances, flow boiling of water on stainless steel.

evaporation [2] or perhaps by allowing K to vary with Jacob number Ja. Nevertheless it seems to work sufficiently well in its simplest form to be taken as a starting point for considering fully-developed nucleate boiling, when the bubble nucleation sites become so numerous that their areas of influence overlap. It has been claimed that the model then still gives satisfactory predictions of the boiling curve, perhaps because the reduction in boiling area due to overlap is compensated by an increase in the convective flux q_c through the stirring action of the bubbles [4]. In this paper we retain all the assumptions of the simple model in order to assess the effect of overlap alone on heat transfer.

Overlap and interference between bubble sites may affect: (a) bubble nucleation, (b) bubble growth and departure, and (c) the rate of heat transfer from the wall. We shall consider (c), assuming that n, f and R_m are known. (a) will be discussed briefly later; little information is yet available about (b).

2. EFFECT OF OVERLAP ON HEAT TRANSFER

Suppose that the fraction of wall area affected by boiling is reduced by overlap from α to $x\alpha$. The consequent reduction in boiling flux is offset by the enhanced rate of heat transfer on the overlapping areas, which are now subjected to quenching by more than one bubble site i.e. at a higher frequency than before. Dividing each area of influence round a bubble site into fractions y_m, where m is the number of influencing sites ($m = 1$ denoting no overlap), then

$$\sum_{m=1}^{\infty} y_m = 1 \tag{5}$$

and averaged over all areas of influence

$$x = \sum_{m=1}^{\infty} y_m/m. \tag{6}$$

Each fraction y_m is quenched m times in the normal bubble period $\tau = 1/f$. With quenching periods of equal duration the increase in mean heat flux would be

$$q_m/q_1 = m^{1/2}. \tag{7}$$

If adjacent sites have the same frequency but are independent in their phase relationships so that the quenching events are randomly distributed over the interval τ, the increase is somewhat smaller

$$q_m/q_1 = \frac{2^{m-1}m!}{1.3\ldots(2m-3)(2m-1)}. \tag{8}$$

The boiling and total fluxes are then found from

$$q_b = q_1\alpha \sum_{m=1}^{\infty} \frac{q_m}{q_1}\frac{y_m}{m} \tag{9}$$

$$q = q_b + q_c\left(1 - \alpha \sum_{m=1}^{\infty} \frac{y_m}{m}\right). \tag{10}$$

3. ESTIMATION OF OVERLAP

The model has been extended beyond the isolated bubble regime only at the expense of introducing further variables y_m which must be determined either from experimental measurements of bubble site positions or from some assumption about their distribution.

Gaertner [5] counted the numbers of nucleation sites on elements of area in low-flux pool boiling ($\alpha < 0.1$) and showed that they conformed to a random (Poisson) distribution. It is difficult to determine the distribution of sites in fully-developed saturated pool boiling because the bubbles obscure the surface. Sites can be detected with an electrical probe such as that described by Iida and Kobayasi [6] but even with similar instrumentation Sultan and Judd [7] found it

Table 1. Overlap parameters for regular 60° array

R/S	$<1/2$	$1/2$	$1/\sqrt{3}$	$\sqrt{3}/2$
α	<0.91	0.91	1.21	2.72
y_1	1	1	0.654	0.027
y_2	0	0	0.346	0.098
y_3	0	0	0	0.875
y_4	0	0	0	0
x	1	1	0.827	0.368
$x\alpha$	α	0.91	1	1
q_b/q_1	α	0.91	1.07	1.52

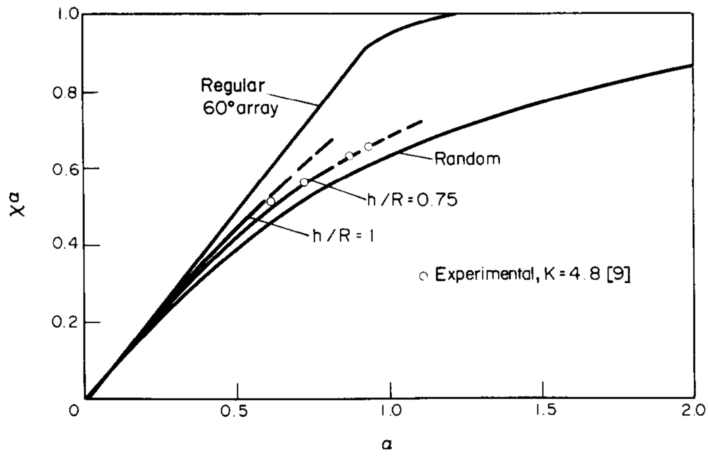

FIG. 2. Effect of overlap on true boiling area.

difficult to distinguish individual sites when their separation decreased towards one bubble diameter. It is easier to identify nucleation sites in flow boiling, particularly when subcooled. Eddington and Kenning [8] found that the potential nucleation sites for water on a stainless steel surface, identified by gas bubble nucleation, were distributed randomly but in low-flux flow boiling experiments on the same surface measurements of nearest-neighbour distances between active sites showed that the probability of nucleation within one bubble departure diameter of another site was greatly diminished. Del Valle M. [9] observed similar behaviour in high-flux flow boiling of water at large subcooling. He showed that the distribution of nearest-neighbour distances gave a more sensitive indication of departures from randomness than Gaertner's method. Again there appeared to be an inhibited zone for nucleation within a distance of about $2R_m$ of an active site (Fig. 1), and some active sites were deactivated by an increase in wall superheat if a new site appeared within this distance. This evidence for an area of influence with $K \approx 4$ in subcooled flow boiling was supported by agreement between the measured heat flux and that predicted by a modified surface-quenching model with $K \approx 5$. However there is no direct evidence for thermal disturbance over regions as large as this in saturated pool boiling.

In the light of the above evidence we first calculate the overlap variables y_m for a random distribution of bubble sites, then consider a distribution modified by interference between sites. Results are also presented for a regular array of active sites on a 60° triangular pitch; the straightforward geometrical calculations are summarized in Table 1.

(*a*) *Random distribution*

The probability that any element da of an area of influence KA forms part of the fraction y_m is the probability that it is overlapped by $(m - 1)$ other areas of influence i.e. that $(m - 1)$ sites lie within a circle of area KA centred on da. The expected number of sites in

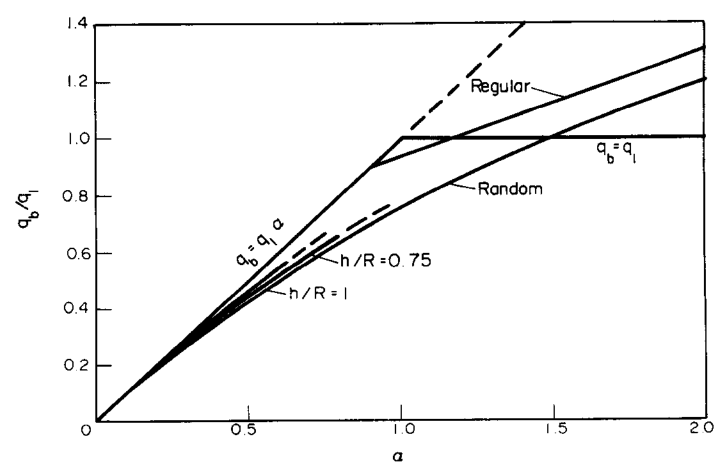

FIG. 3. Effect of overlap on boiling heat flux.

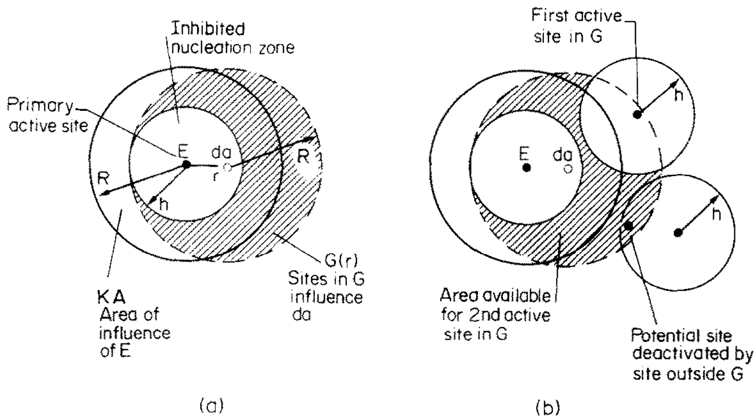

FIG. 4. Model for calculation of overlap with inhibited nucleation zone.

KA is $nKA = \alpha$, so for a Poisson distribution the probability of having $(m - 1)$ sites is

$$P(m-1) \equiv y_m = \frac{e^{-\alpha}\alpha^{m-1}}{(m-1)!}. \quad (11)$$

Substituting into equation (6), the fraction of wall area affected by boiling is given by

$$x\alpha = (1 - e^{-\alpha}). \quad (12)$$

This prediction is plotted in Fig. 2 together with $x\alpha$ for a regular 60° array. Also shown are values deduced from the active site positions and bubble radii measured by cine photography in [9]: the corresponding areas of influence (taking $K = 4.8$) were plotted to give the true boiling areas. The experimental points lie between the theoretical lines. This is consistent with distortion of the random distribution by a bias against nucleation close to another site.

From equations (8), (9) and (11) the boiling heat flux for a random site distribution is given by

$$q_b = q_1 \alpha\, e^{-\alpha} \sum_{m=1}^{\infty} \frac{(2\alpha)^{m-1}}{1.3\ldots(2m-1)}$$

$$= q_1 \frac{(\pi\alpha)^{1/2}}{2} \operatorname{erf} \alpha^{1/2} \quad (13)$$

using an expansion for $\operatorname{erf}\alpha^{1/2}$ from [10]. The boiling heat flux normalized with respect to the quenching flux q_1 is plotted in Fig. 3 for the random distribution, equation (13), also the regular distribution with equation (8). If no allowance were made for heat flux enhancement, the boiling flux would be bounded by $q_b = q_1\alpha = q_{bi}$ for $\alpha \leqslant 1$, $q_b = q_1$ for $\alpha \geqslant 1$. At $\alpha = 1$ the flux with a random distribution is 25% less than the flux ignoring overlap, but for $\alpha > 1.5$ the flux exceeds q_1 as the assumed enhancement on overlapping regions becomes increasingly important. The heat flux with regular site distribution is somewhat higher, with no effects of overlap for $\alpha < 0.9$.

(b) Modified random distribution

From the experimental evidence there is a reduced probability of activation of two sites in close proximity, for which we make no attempt here to identify the cause. We simply make the crude assumption that there is no nucleation at all within a distance h of an active site (i.e. in an area πh^2) and no effect on nucleation beyond this distance. This is equivalent to a sharp cut-off at $s = h$ in the distribution of nearest-neighbour distances s, which for a Poisson site distribution is [5]

$$P(s, s + ds) = 2\pi n_0 s\, e^{-n_0\pi s^2}\, ds \quad (14)$$

n_0 is the density of *potential** nucleation sites, which are distributed randomly. Thus the mean density of *active* sites n is given approximately by

$$n_0 \int_h^{\infty} P(s)\, ds$$

whence

$$\frac{n}{n_0} \approx e^{-n_0\pi h^2} \quad \text{or} \quad \frac{\alpha}{\alpha_0} \approx e^{-\alpha_0 (h/R)^2} \quad (15)$$

where

$$\alpha_0 = n_0 KA = n_0 \pi R^2, \quad R^2 = K R_m^2. \quad (16)$$

It is shown in Appendix I that equation (15) underestimates n/n_0 and a correction is proposed.

In order to determine y_m we again consider an element da of an area of influence KA, but this time at a specified radius r from the primary site E, Fig. 4. Part of the area KA round da in which other influencing sites may occur now falls within the inhibited zone round E, causing a reduction in area to $G(r)$. y_m depends on the probability $P(m - 1)$ of having $(m - 1)$ active sites in

*In this context a potential site is one which, at the prevailing temperature and pressure, would be active in the absence of other sites. It does not mean a liquid-filled cavity which only becomes active after seeding by vapour from another site.

G; if there is one active site in G its inhibited zone reduces still further the area available for additional sites. In Appendix II we develop an approximate method for the numerical evaluation of $P(m - 1)$. Then y_m is found from

$$y_m = \int_0^R P(m-1)\frac{2r}{R^2}\,dr \qquad (17)$$

for use in equation (6) and equation (9) with (8). The values of true boiling area $x\alpha$ deduced from the experiments in [9] lie close to the resulting calculations for $0.75 \leqslant h/R \leqslant 1$ (Fig. 2). The calculated heat flux q_b lies between the lines for the random and regular site distributions (Fig. 3).

4. DISCUSSION

(a) *Heat transfer model*

In the surface-quenching model it is assumed that quenching is uniform up to the edge of the area of influence: this leads to the model for enhanced heat flux on overlapping areas used in this paper. The experimental evidence for suppression of nucleation over distances similar to the radii of areas of influence deduced from heat transfer calculations provides some support for this model but there is no supporting evidence from local measurements of heat flux near growing bubbles. If the heat flux is in fact very intense near the bubble but decays rapidly then the effect of overlap would be reduced.

It has been shown that overlap does not greatly affect the boiling heat flux *for a given density of active sites*, whatever the details of their distribution. Heat flux differences not exceeding 25% between regular and random distributions are small compared with the uncertainties in other variables incorporated in the model, such as bubble size and frequency, and represent only a small error in wall superheat. A contribution to the boiling flux by microlayer evaporation would further reduce the difference between the isolated and overlapping predictions. (Latent heat transport occurs only after heat transfer from the wall to the liquid so it does not provide a further contribution.)

The model for the boiling heat flux is not yet a predictive tool since at any wall superheat ΔT we need as input the active site density n and the bubble parameters, which may depend on n as well as ΔT.

(b) *Nucleation site density*

With further development it may become possible to estimate the density of potential sites n_0 from experiments with supersaturated gas solutions [1, 4, 8]. It is in determining the corresponding active site density n that the precise nature of the processes inhibiting nucleation can influence the boiling curve.

The increased quenching frequency in overlapping regions subjects potential nuclei to larger temperature gradients so suppression on at least part of an area of influence is consistent with Hsu's nucleation model [11]. From Figs. 1 and 2 we expect h/R for our crude suppression model to be in the range $0.75 \leqslant h/R \leqslant 1$. From Appendix I and Fig. A2 the active site density n is sensitive to the value of h/R, which is based on very limited evidence from subcooled flow boiling of water on relatively smooth surfaces. A much more extensive investigation of nucleation interference is required. A further complication may be the activation of unstable sites by vapour from stable sites. This occurred to a very limited extent for water on stainless steel [8] but may be much more important for well-wetting organic and cryogenic liquids. However, whatever the initial process of activation, the inhibiting mechanisms should apply once regular bubble production at a site has been established.

(c) *Bubble coalescence*

An essential feature of the surface-quenching boiling model is that bubbles retain their identities from nucleation to departure or collapse. If bubble coalescence occurs on the heated surface (as opposed to in the bulk liquid) an entirely different model must be applied to those areas covered by vapour patches, eventually leading to a prediction of the critical heat flux.

Hsu [12] analysed coalescence for a random site distribution by considering the overlap of time-averaged bubble areas for the instantaneous bubble population. Agreement was claimed with saturated pool boiling experiments within the large statistical uncertainty arising from the small heated area and the consequent small population of active nucleation sites. An unsatisfactory feature of the experiments was the small width of the heated strip which was little more than the bubble diameter, distorting the two-dimensional nature of coalescence. Bald [13] suggested a simple coalescence criterion based on bubble departure size and the mean nearest-neighbour distance. Neither of these approaches considers in detail the separation of bubbles in time, as well as space, which is a feature of subcooled boiling [9], and both assume a random site distribution. We can expect bubble coalescence to be extremely sensitive to departures from randomness: if $h/R \approx 1$ and $K \geqslant 4$ there should be no coalescence at all! We know that coalescence does occur so a better understanding is required of the mechanisms by which nucleation is inhibited and the consequences for coalescence. In this paper we have ignored the statistical variations in bubble size and frequency, which may be important.

5. CONCLUSION

The quenching model for nucleate boiling heat transfer has been extended to high bubble site densities by assuming an enhanced rate of heat transfer on those parts of the surface overlapped by several areas of influence. The effect of overlap on heat transfer for a given density of active sites is not large: a more important limitation lies in the uncertainty in the correlations for bubble size and frequency. If the model is valid at low heat flux, it should remain valid with increasing flux until bubbles no longer retain individual identities on the surface.

Subcooled flow boiling experiments provide limited evidence for the inhibition of nucleation within an area of influence of about two bubble radii round an active site, which distorts the random distribution of the sites. Further work on the mechanism of inhibition is required for the prediction of (a) active site density and (b) bubble coalescence, which eventually limits the application of the quenching model.

Acknowledgements—Victor Del Valle M. was supported by the Consejo Nacional de Ciencia y Tecnologia, Mexico and the photographic study of subcooled flow boiling was funded by the U.K. Atomic Energy Authority, Harwell.

REFERENCES

1. J. J. Lorenz, B. B. Mikic and W. M. Rohsenow, The effect of surface conditions on boiling characteristics, *Proc. 5th Int. Heat Transfer Conf.* (Tokyo), IV, 35–39 (1974).
2. R. L. Judd and K. S. Hwang, A comprehensive model for nucleate pool boiling heat transfer including microlayer evaporation, *J. Heat Transfer* **98**, 623–629 (1976).
3. K. Bier, D. Gorenflo, M. Salem and Y. Tanes, Pool boiling heat transfer and size of active nucleation centres for horizontal plates with different surface roughness, *Proc 6th Int. Heat Transfer Conf.* (Toronto), I, 151–156 (1978).
4. A. Singh, B. B. Mikic and W. M. Rohsenow, Relative behaviour of water and organics in boiling, *Proc.* 6th *Int. Heat Transfer Conf.* (Toronto), I, 163–168 (1978).
5. R. F. Gaertner, Distribution of active sites in the nucleate boiling of liquids, *Chemical Engineering Progress Symposium Series* No. 41, **59**, 52–61 (1963).
6. Y. Iida and K. Kobayasi, An experimental investigation on the mechanism of pool boiling phenomena by a probe method, *Proc. 4th Int. Heat Transfer Conf.* (Versailles), V, B1.3 (1970).
7. M. Sultan and R. L. Judd, Spatial distribution of active sites and bubble flux density, *J. Heat Transfer* **100**, 56–62 (1978).
8. R. I. Eddington and D. B. R. Kenning, The prediction of flow boiling bubble populations from gas bubble nucleation experiments, *Proc. 6th Int. Heat Transfer Conf.* (Toronto), I, 275–280 (1978).
9. V. H. Del Valle M., D.Phil thesis, Department of Engineering Science, Oxford University (1980).
10. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, p. 297. Dover, New York (1965).
11. Y. Y. Hsu, On the size range of active nucleation cavities on a heating surface. *Trans. A.S.M.E. J. Heat Transfer* **C84**, 207–216 (1962).
12. Y. Y. Hsu, Gradual transition of nucleate boiling from discrete-bubble regime to multibubble regime. NASA TN D-2564 (1964).
13. W. B. Bald, Cryogenic heat transfer research at Oxford, *Cryogenics* **13**, 457–469 (1973).

APPENDIX I

INHIBITED NUCLEATION ZONE: EFFECT ON POPULATION

Suppose all the potential nucleation sites capable of activation at a given superheat follow a Poisson distribution with mean density n_0. The distribution of nearest-neighbour distances is [equation (14)]

$$P(s, s+\mathrm{d}s) = 2\pi n_0 s\,\mathrm{e}^{-n_0\pi s^2}\,\mathrm{d}s \tag{A1}$$

with most probable value

$$S_{mp} = (2\pi n_0)^{-1/2}. \tag{A2}$$

Next impose the condition that an active site must be a distance $\geqslant h$ from any other active site, $P(s < h) = 0$, by an elimination process starting at $s = 0$ and proceeding to $s = h$. Each elimination removes one of a pair of sites: if no other sites were affected the new site density would be

$$n_0 \int_h^\infty P(s)\,\mathrm{d}s$$

whence

$$\frac{n}{n_0} = \frac{\alpha}{\alpha_0} = \mathrm{e}^{-n_0\pi h^2} = \mathrm{e}^{-\alpha_0(h/R)^2}. \tag{A3}$$

However the eliminated site of a pair at distance s may also have been nearest neighbour to another site at $s' > s$, which is then moved into a group with still larger $s'' > s'$. Consequently some sites are transferred from $s < h$ to $s > h$, which has the effect, but in a way that is not readily calculated, of increasing

$$\int_h^\infty P(s)\,\mathrm{d}s.$$

A further indication that equation (A3) underestimates the active site density is its prediction of maxima in n and α at $\alpha_0 = (R/h)^2$

$$\alpha_{\max} = \frac{1}{\mathrm{e}}\left(\frac{R}{h}\right)^2. \tag{A4}$$

This is contrary to physical intuition which suggests that at large α_0 n and α should approach constant values less than the maximum values for a regular array of sites at the centres of close-packed circles of radius $h/2$ (i.e. with $s = h$ for all sites)

$$\alpha_{\max}^{\mathrm{reg}} = \frac{2\pi}{\sqrt{3}}\left(\frac{R}{h}\right)^2. \tag{A5}$$

The physical basis for the inhibition model is insufficient to warrant an elaborate analysis so we shall use simple arguments to obtain upper and lower bounds for the active site density and thus improve on equation (A3).

Consider a potential site E which is active provided no other active site lies inside the circle of radius h about E, Fig. A1. The probability P_A that E is active may therefore be written as

$$\begin{aligned} P_A = \frac{n}{n_0} = \frac{\alpha}{\alpha_0} &= \sum_{m=0}^{\infty} P(m)\,\bar{P}_D^m \\ &= \mathrm{e}^{-n_0\pi h^2(1-\bar{P}_D)} \\ &= \mathrm{e}^{-\alpha_0(h/R)^2(1-\bar{P}_D)} \end{aligned} \tag{A6}$$

where $P(m)$ is the Poisson probability of finding m potential sites inside h, $\bar{P}_D$ is the probability that such a site F is deactivated by a site outside h. (If the deactivating site were inside h it would necessarily deactivate E also.) For a site F the survival probability $P_s = 1 - \bar{P}_D$ is the probability that there are no active sites in the shaded area A_F, (Fig. A1). P_s

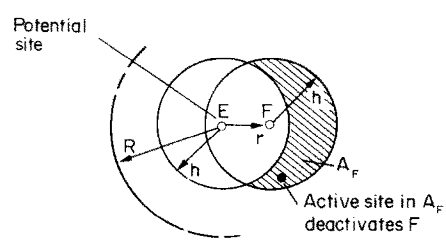

FIG. A1. Model for interference between sites.

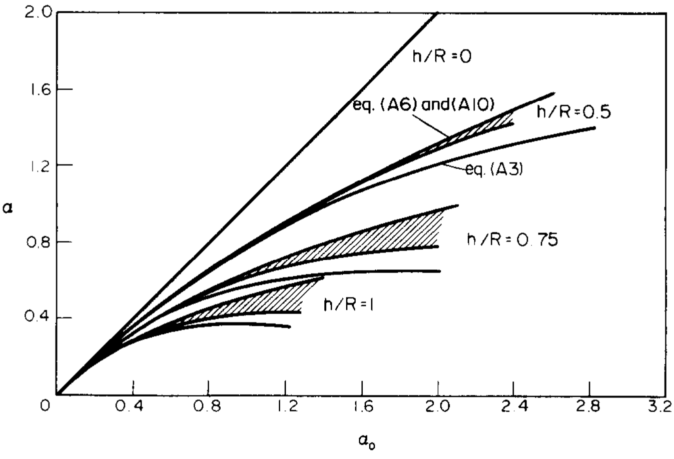

FIG. A2. Effect of inhibited zone on active site population.

depends on the radial distance of F from E but since the potential sites are randomly distributed we use in equation (A6) the mean value of P_s over the area πh^2 round E

$$\bar{P}_s = 1 - \bar{P}_D = \int_0^h \frac{2r}{h} P_s(r)\,\mathrm{d}r. \qquad \text{(A7)}$$

The potential sites in A_F are themselves subject to deactivation but they are partially shielded by the known absence of active sites in the adjacent area πh^2. Thus their probability of activity must lie between unity and P_A, the value for the population at large, setting limits on P_s

$$\mathrm{e}^{-n_0 A_F} < P_s < \mathrm{e}^{-n A_F}. \qquad \text{(A8)}$$

Conveniently A_F varies almost linearly with r

$$A_F \approx 0.61\pi h^2(r/h). \qquad \text{(A9)}$$

From equations (A7), (A8) and (A9)

$$\bar{P}_s = \frac{5.37}{\boldsymbol{\alpha}^2(h/R)^4}\left[1 - (1 + 0.61\boldsymbol{\alpha}(h/R)^2)\mathrm{e}^{-0.61\boldsymbol{\alpha}(h/R)^2}\right] \qquad \text{(A10)}$$

where $\boldsymbol{\alpha} = \alpha$ or α_0. The bounds for $\alpha(\alpha_0, h/R)$ from equations (A6) with (A10) are plotted in Fig. A2, together with equation (A3). The lower bound still reaches a maximum at a finite value of α_0 while the upper bound increases without limit so the present theory is inadequate to predict the limiting values of α. Nevertheless the uncertainty is sufficiently small up to $\alpha_0 \approx (R/h)^2$ to demonstrate the sensitivity of the active site density n and the nominal boiling area α to the precise value of the nucleation inhibition parameter h/R.

APPENDIX II

INHIBITED NUCLEATION ZONE: EFFECT ON OVERLAP AND BOILING HEAT FLUX

In Fig. 4 the element $\mathrm{d}a$ of the area of influence at distance r from the active site E is part of the fraction y_m if $(m - 1)$ sites are active in the area $G(r)$ so we have to estimate the probability $P(j)$ of j active sites in G given the Poisson probability $P(k)$ of k potential sites, a more complicated calculation than was required in Appendix I. The procedure and simplifying assumptions are given in outline only.

If $j = 0$ all the potential sites must be deactivated by sites outside G, for which the probability is $P_D = 1 - P_s$. Then

$$P(0) = \sum_{k=0}^{\infty} \mathrm{e}^{-n_0 G}\frac{(n_0 G)^k}{k!}(1 - P_s)^k = \mathrm{e}^{-n_0 G P_s}. \qquad \text{(A11)}$$

If $j = 1$ the active site can be chosen in k ways and is surrounded by an inhibited zone covering a fraction ϕ_1 of G. The chosen site has only to survive deactivation by sites outside G, survival probability P_s. Each of the remaining $k - 1$ sites must be inactive, either because it lies in the inhibited zone of the active site (probability ϕ_1 since potential sites are distributed randomly) or through deactivation by sites outside G if it lies outside the inhibited zone (probability $(1 - \phi_1)(1 - P_s)$. Hence

$$P(1) = \sum_{k=1}^{\infty} \mathrm{e}^{-n_0 G}\frac{(n_0 G)^k}{k!} k P_s[1 - (1 - \phi_1)P_s]^{k-1}$$
$$= n_0 G P_s \mathrm{e}^{-n_0 G P_s(1-\phi_1)}. \qquad \text{(A12)}$$

In general, j active sites can be chosen in $k!/(k - j)!j!$ ways. The first site can be deactivated only by sites outside G, survival probability P_s. The second site must lie outside the inhibited zone of the first site and also survive the effect of external sites, probability $(1 - \phi_1)P_s$. The jth site must lie outside the inhibited zones of all the preceding sites, in area $G(1 - \phi_{j-1})$ say, probability of survival $(1 - \phi_{j-1})P_s$. The remaining $k - j$ sites must be inactive, probability $1 - (1 - \phi_j)P_s$.

$$\therefore P(j) = \sum_{k=j}^{\infty} \mathrm{e}^{-n_0 G}\frac{(n_0 G)^k}{k!}\frac{k!}{(k-j)!j!}$$
$$\times P_s^j(1-\phi_1)\ldots(1-\phi_{j-1})[1-(1-\phi_j)P_s]^{k-j}$$
$$= \frac{(n_0 G P_s)^j}{j!}(1-\phi_1)\ldots(1-\phi_{j-1})\mathrm{e}^{-n_0 G P_s(1-\phi_j)}. \qquad \text{(A13)}$$

In equations (A11)–(A13) we have taken P_s to be constant, neglecting any dependence on j. P_s must depend on the shape as well as the size of G but for simplicity we consider a circle of the same area as G and calculate P_s as in Appendix I, assuming all potential sites outside G to be active [cf. equation (A8)]. Equation A9 does not apply since the radii of the intersecting circles are no longer equal, so the integral corresponding to equation (A7) is evaluated numerically.

The area of inhibited nucleation round an active site in G may lie partly outside G so the probabilities ϕ_1 etc. are also calculated for equivalent circles, assuming equal probability of finding the site anywhere in the circle.

With the above P_s and ϕ_j, $P(j)$ is calculated starting at $j = 0$ and proceeding until $\Sigma P(j) > 1$ (which may occur due to the approximate nature of the calculations), or until $j = 5$. The last value is then corrected, or $P(6)$ chosen, to satisfy $\Sigma P(j) = 1$.

The procedure is repeated for intervals of $0.1R$ over $0 \leqslant r \leqslant R$ to give mean values for $P(j)$ over the area of influence and hence y_m for substitution in equation (6) for boiling area reduction x and equation (9) for boiling flux.

The calculations for $h/R = 0.75$ and 1 are shown in Figs. 2 and 3. The range of validity is limited by the uncertainty in $\alpha(\alpha_0, h/R)$, Appendix I. A more profitable approach might be a numerical experiment with a random array of potential sites on a computer but this does not seem worth undertaking until we have better models for heat transfer near an active site and for inhibition of nucleation.

EBULLITION NUCLEE PLEINEMENT DEVELOPPEE: RECOUVREMENT DES AIRES D'INFLUENCE ET INTERFERENCE ENTRE SITES DE NUCLEATION

Résumé—Le modèle de trempe de surface pour le transfert thermique par ébullition à faible densité de sites de nucléation a été étendu aux fortes densités. L'accroissement du flux de chaleur dû à l'augmentation des fréquences de trempe est contrebalancé par la réduction des aires d'ébullition, là où les aires d'influence de plusieurs sites de nucléation se recouvrent partiellement. Le degré de recouvrement est estimé pour des distributions régulières et au hasard des sites de nucléation, et pour une distribution au hasard modifiée par une interférence à courte distance entre sites. Son effet n'est pas important sur le transfert de chaleur, dans le domaine d'intérêt pratique.

On révise l'aspect de l'inhibition de la nucléation par l'interférence thermique des sites actifs. Bien que l'interférence ait un faible effet sur le modèle de transfert de chaleur pour un nombre spécifié de sites actifs, elle peut influencer la courbe d'ébullition en modifiant le nombre des sites actifs et la formation de réseau de vapeur par coalescence de bulles. On obtient des relations entre les densités de potentiel et les sites actifs.

VOLLENTWICKELTES BLASENSIEDEN: ÜBERSCHNEIDUNG DER EINFLUSSGEBIETE UND INTERFERENZ VON BLASENKEIMEN

Zusammenfassung—Das Oberflächenabkühlungsmodell für den Wärmeübergang beim Sieden bei kleinen Keimstellendichten wurde auf hohe Dichten ausgedehnt. Der verbesserte Wärmeübergang aufgrund der erhöhten Abkühlfrequenz in den Überschneidungsgebieten verschiedener Keimstellen gleicht die Verkleinerung der Siedefläche teilweise aus. Für Zufallsverteilungen und für gleichmäßige Anordnungen von Keimstellen wurde der Grad der Überschneidung abgeschätzt und für eine Zufallsverteilung durch eine Nahinterferenz zwischen den Keimen modifiziert. Der Einfluß auf den Wärmeübergang im praktisch interessierenden Bereich ist nicht groß.

Das Auftreten von Keimbildungsunterdrückung durch thermische Interferenz seitens aktiver Keimstellen wird diskutiert. Obwohl Interferenz für eine bestimmte Anzahl aktiver Keime nur geringen Einfluß auf das Wärmeübergangsmodell hat, kann sie sich im Bereich des entwickelten Siedens durch Beeinflussung der Zahl aktiver Keime und die Bildung von Dampfbereichen durch Zusammenwachsen von Blasen auswirken. Beziehungen zwischen den potentiellen und aktiven Keimstellendichten wurden abgeleitet.

ПОЛНОСТЬЮ РАЗВИТОЕ ПУЗЫРЬКОВОЕ КИПЕНИЕ. ВОЗДЕЙСТВИЕ И ВЗАИМОВЛИЯНИЕ ЦЕНТРОВ ОБРАЗОВАНИЯ ПУЗЫРЬКОВ

Аннотация — Модель резкого охлаждения поверхности, обычно применяемая в исследованиях теплопереноса при кипении в случае небольшой плотности центров образования пузырьков, использована для случая их большой плотности. При этом рост интенсивности переноса тепла с увеличением частоты охлаждения, когда происходит наложение областей взаимного влияния нескольких центров, компенсирует уменьшение площади поверхности кипения. Проведена оценка «степени» наложения для неупорядоченных и упорядоченных распределений центров и для случая неупорядоченного распределения при ближнем порядке и их взаимном влиянии друг на друга. Оказывается, что влияние степени перекрытия областей на процесс теплопереноса не существенно с практической точки зрения. Рассматривается вопрос о торможении процесса образования пузырьков тепловым воздействием от активных центров парообразования. Хотя это воздействие и не оказывает большого влияния на теплоперенос при фиксированном числе активных центров, оно все же влияет на развитое кипение за счет изменения числа активных центров и образования участков, содержащих пар в результате слияния пузырьков. Выведены соотношения между плотностями потенциальных и активных центров парообразования.